Universal Lower Bounds on Energy — Computational Aspects

Abstract

Let $S^{n-1}$ be the unit sphere in $\mathbb{R}^n$. We refer to a finite set $C \subset S^{n-1}$ as a spherical code and, for a given (extended real-valued) function $h(t) : [-1, 1] \rightarrow [0, +\infty]$, we consider the $h$-energy (or the potential energy) of $C$ defined by

$$E(n, C; h) := \sum_{x, y \in C, x \neq y} h(\langle x, y \rangle),$$

where $\langle x, y \rangle$ denotes the inner product of $x$ and $y$.

A commonly arising problem is to estimate the potential energy provided the cardinality $|C|$ of $C$ is fixed, that is, to determine

$$\mathcal{E}(n, N; h) := \inf\{E(C; h) : |C| = N, C \subset S^{n-1}\}$$

In this talk we address some computational aspects in connection with recently obtained universal lower bound on $\mathcal{E}(n, N; h)$ — decision making, generating necessary parameters, deriving bounds, test functions and (if possible) further improvements.

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Noon – 1:00, Wednesday, March 18, 2015. Location: KT 216