THE DEPARTMENT OF MATHEMATICAL SCIENCES

Indiana University - Purdue University Fort Wayne

is pleased to present

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Efficient Spherical Designs with Good Geometric Properties

Abstract
Spherical $t$-designs on the unit sphere $S^d \subset \mathbb{R}^{d+1}$, introduced by Delsarte, Goethals, and Seidel (1977), are equal weight numerical integration rules that are exact for all polynomials of degree at most $t$ on $S^d$. This talk considers the calculation and properties of spherical $t$-designs, in particular for $S^2$ where most applications reside.

Bondarenko, Radchenko, and Viazovska (2013) proved that there exists a $c_d$ such that spherical $t$-designs with $N$ points exist for all $N \geq c_d t^d$, which is the optimal order. Moreover they showed that there exist such spherical designs that are well-separated (2014). The interest here is in efficient spherical designs with $N < t^d$.

The geometric properties of point sets on $S^d$ can be characterised by their separation (twice the packing radius), their mesh norm (covering radius), and mesh ratio (covering radius / packing radius), amongst many other criteria. A common assumption arising in applications is that the the sequence of point sets is quasi-uniform, that is, their mesh ratios are uniformly bounded. The interest here is in sets of efficient spherical $t$-designs with small mesh ratios.

Examples of spherical $t$-designs on $S^2$ with $N = t^2/2 + O(t)$ points and mesh ratio $< 1.8$ for $t = 1, \ldots, 311$ are available from:
http://www.maths.unsw.edu.au/~rsw/Sphere/EffSphDes/

These provide excellent sets of points for both numerical integration and approximation, for example by needlets.

Noon – 1:00, Wednesday, December 2, 2015. Location: KT 216